REMARKS ON C. LLOYD MORGAN'S PAPER—"RELATION OF STIMULUS TO SENSATION."[1]

By MAX MEYER.

Morgan reports in the above mentioned paper on some interesting experiments, from which he draws the conclusion, that Weber's law does not hold good with visual sensations. This important conclusion is based on the mathematical discussion of the results of his experiments. This mathematical discussion, however, contains several errors, to which I wish to call attention. As soon as these are corrected, there is no argument left in favor of Morgan's conclusion. His experiments, far from contradicting Weber's law, confirm it.

Morgan mixes white with black, red with black, and blue with black in such a manner, that on the rotating disc a smooth and even grading from the center to the periphery results. The distribution of white, red and blue in each case is represented by the curves in Fig. 2 on page 225 of his paper. The author asserts "that neither of these curves is throughout its whole extent logarithmic as it should be, if the Weber-Fechner formula holds good." This, indeed, would be a surprising result, if it were true. He has, however, found another law in his experiments, namely that, "*Equal increments of sensation are produced by increments of excitation in geometrical progression.*" The mathematicians would be interested to know the difference between such a curve and a logarithmic curve. Obviously Morgan does not realize, that just this is the specific property of a *logarithmic* curve, viz.: that the increments of one co-ordinate form a geometrical series, when the increments of the other co-ordinate form an arithmetical one. His assertion that his curves are not logarithmic, is the more wonderful when one notices that his colleague Barrel, in the note on page 228, determines the equation of the curves as

$$x = A\ (10^{by} - 1),$$

which represents just that kind of curve.

In Fig. 4 Morgan compares two curves, a continuous line curve and a broken line curve. The equation of the continuous line curve is given. The equation of the broken line curve Morgan does not give, but states simply that: "The broken line curve

[1] Psychological Review VII, No. 3, pp. 217-233. 1900.

shows *the logarithmic curve*, which passes through the percentages at stages 6 and 14." Yet no curve is determined by two given points. Through two points not only one logarithmic curve can be drawn, but infinitely many. One of them is the continuous curve. What Morgan means by saying that the broken line "is one of the best logarithmic curves which can be found for purposes of comparison," no one can tell. The "best" logarithmic curve is certainly that logarithmic curve, which represents the observed facts, *i. e.*, the continuous line itself.

Yet it is not a merely mathematical misconception which underlies this confusion. On page 232 Morgan says of his black background, that this black "may be regarded as incapable of affording any appreciable amount of positive stimulation to the retina." This is doubtless a serious mistake. There is no reason of presuming, that his "black" can be represented in the equation by the stimulus $x=0$. On page 226 he says: "In the curves plotted in Fig. 2 the stimuli required to produce the sensation series 5%, 10%, 15%, etc., are *not* in geometrical progression." He does not see, that what he calls stimuli (the figures 3.49, 7.74, 12.94, etc., in Table III) are not the stimuli, but the differences between the stimuli and the constant A. A in the case of "white on black" is equal 15.85 (see note p. 228). If he had added to each of the above figures 15.85, he would have found, that the stimuli are actually in geometrical progression.

In comparing the curves for white, red and blue, Morgan construes these curves in Fig. 2 in such a manner, that comparison is quite impossible. If we wish to compare these three logarithmic curves, the best method is, of course, to represent all of them as parts of one logarithmic curve, the constants of which we choose arbitrarily, in such a manner that the points representing "black" (*i. e.*, Morgan's black, not the absolute black) are identical.

We may choose as constants of an arbitrary logarithmic curve, the equation of which is $x=A\,(10^{by}-1)$,

$$A=15.85 \quad , b=0.008639 \quad .$$

A and b are then identical with A and b for "white on black" in Morgan's paper. This curve is then identical with Morgan's curve for "white on black" in Fig. 2. The ends of the curve represent "black" and "white." We have still to determine, which points of this curve represent "red" and "blue."

We make use, in the case of "red," of the substitutions

$$x=p'\,x',\ y=g'\,y'.$$
$$\therefore\ p'\,x'=A\,(10^{bg'y'}-1)$$

The conditions for this being the equation of "red on black" are

$$x'=35, \text{ when } y'=50$$
$$x'=100, \text{ when } y'=100$$
$$\therefore 35\ p'=A\ (10^{50bg'}-1)$$
and
$$100\ p'=A\ (10^{100bg'}-1)$$
$$\therefore \frac{100}{35}=\frac{A\ (10^{50bg'}-1)(10^{50bg'}+1)}{A\ (10^{50bg'}-1)}$$

$$\frac{100}{35}=10^{50bg'}+1$$

$$50\ bg'=\log\left(\frac{100}{35}-1\right)=\log 13-\log 7$$
$$g'=\frac{1}{50b}\ (\log 13-\log 7)$$
$$g'=0.621$$

Consequently Morgan's "red" must be represented in our curve by $y=100\ g'=62.1$.

The quantity of p' needs not be calculated, since the point of "red" on our curve is already determined by y alone.

For "blue on black" we use the substitutions

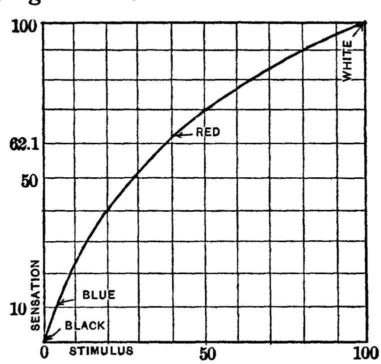
$$x=p''\ x'',\ y=g''\ y''.$$

Conditions for "blue on black":

$$x''=47.5, \text{ when } y''=50$$
$$x''=100, \text{ when } y''=100$$
$$\therefore\ 10^{50bg''}=\frac{100}{47.5}-1=\frac{52.5}{47.5}=\frac{21}{19}$$

$$g''=\frac{1}{50b}\ (\log 21-\log 19)$$
$$g''=0.100$$

Consequently Morgan's "blue" must be represented in our curve by $y=100\ g''=10.0$.



The figure shows that the distance from "white" to "black" is considerably greater than from "red" to "black" and the latter again greater than from "blue" to "black." This is easily understood, since Morgan's blue was probably darker than his red and this darker than his white. If we knew that the relative intensities of these three stimuli were $(A+100\ p''):(A+100\ p'):(A+100)$ we could draw the conclusion, that the judgment in each of the three cases were conditioned simply by the intensities of light, the redness and blueness being of no consequence. However, Morgan makes no statement with respect to the relative intensities of his "white," "red" and "blue."